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THE LOSS PARAMETERS FOR VERY SHORT BUNCHES

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Abstract

Semi-empirical formulas for the transverse and longitudinal loss factors generated by cavity and step discontinuities are given in the limit of short bunch length. The parametric transition between the cavity and step approximations is considered. The differences between the impedances offered by periodic structures and isolated single cavities are also discussed.

The short bunch lengths common today in designs of linear colliders, FEL drivers, and synchrotron light sources require a thorough evaluation of the loss factors generated by various machine components. However, the estimation of the loss factors with available numerical codes such as TBCI is not straightforward because of the limitation on the number of mesh points, which becomes very large for short bunches.

The main contribution to the loss factors of typical machines is given by elements of the system which can be approximately described as pill-box cavities with attached tubes or as discontinuities of the beam-pipe cross-section due to an abrupt change of radius. We refer to these two basic elements as a cavity and a step. In this note we give handy analytic expressions for loss parameters for these two cases. We compare them with results of numerical simulations using TBCI and give the range of their applicability. At the end we discuss how tapering reduces the loss factor of a step. For convenience we start the paper with definitions and some basic formulas.

The longitudinal δ-function wake W_l^δ, by definition[1],[2], gives the energy loss ΔE_1 of a particle that follows a point-like bunch with the total charge $Q = eN_b$ at the distance s

$$\Delta E_1(s) = -eQW_l^\delta(s), \quad s > 0 \tag{1}$$

The average energy loss k_l of a particle in a bunch with longitudinal density $\rho(z)$ normalized to unity is

$$k_l = -< \frac{\Delta E_1}{eQ} >= \int\limits_{-\infty}^{\infty} dz_1 \int\limits_{-\infty}^{z_1} dz_2 \rho(z_1)\rho(z_2)W_l^\delta(z_1 - z_2) \tag{2}$$

Fourier transformation of a function F(x) is defined as

$$\tilde{F}(k) = \int\limits_{-\infty}^{\infty} dx F(x) e^{ikx}$$

For a Gaussian bunch with $< x^2 >= \sigma^2$, the density

$$\rho(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-z^2/2\sigma^2}$$

has Fourier harmonics

$$\tilde{\rho}(k) = e^{-k^2\sigma^2/2}$$

and Eq. (2) can be rewritten as

$$< \Delta E_1 >= -\frac{eQ}{2\pi}\int_{-\infty}^{\infty} dk\tilde{W}^s(k)|\tilde{\rho}(k)|^2$$

W_l^s can be related to the longitudinal component of electric wake field $\vec{E}$ coherently excited by particles of a bunch through the expression

$$\Delta E_1(s) = -eQW_l^s(s) = e\int dzE_z(\vec{r}, z, t = (z+s)/v). \tag{3}$$

The longitudinal impedance $Z_l(\omega)$ is defined as the voltage induced by a periodic current with frequency ω divided by the amplitude of the current. By definition

$$Z_l(\omega) = \frac{1}{Q}\int dz\tilde{E}_z(\vec{r}, z, k)e^{-i\omega z/v} \tag{4}$$

The integral in Eqs. (3) and (4) does not depend on the choice of the transverse coordinate $\vec{r}$ if the wake generating element is axially symmetric. The usual choices are r set equal to 0 with integration from $-\infty$ to ∞ or, more conveniently, r set equal to the beam pipe radius.

Comparison of Eqs. (3) and (4) gives the impedance as Fourier harmonics of the wake:

$$Z_l(\omega) = \tilde{W}_l^s(\omega/v)/v \tag{5}$$

For small losses, the impedance can be expressed as a sum over modes with frequencies ω_λ, widths γ_λ and loss-factors χ_λ

$$Z_l(\omega) = i \sum_\lambda \chi_\lambda \left(\frac{1}{\omega - \omega_\lambda + i\gamma_\lambda} + \frac{1}{\omega + \omega_\lambda + i\gamma_\lambda} \right) \tag{6}$$

According to Eq. (5), the wake in this case is given as

$$W_l^\delta(s) = 2 \sum_\lambda \chi_\lambda \cos\left(\frac{\omega_\lambda s}{v}\right) e^{-\gamma_\lambda s/v} \tag{7}$$

In Eq. (7) we have assumed that $s > 0$. For $s = 0$,

$$W_l^\delta(s) = \sum_\lambda \chi_\lambda \, .$$

In terms of the loss parameter, the energy loss of a particle is

$$\Delta E_\lambda = e^2 N_B \chi_\lambda$$

A loss parameter can be found, if the eigenfunction of the mode E_z^λ are known

$$\chi_\lambda = \frac{|V_\lambda|^2}{4U_\lambda}, \tag{8}$$

where

$$V_\lambda = \int dz e^{-ikz} E_z^\lambda(\vec{r}, z) \tag{9}$$

and U_λ is the energy, stored in a mode. The mode loss factor χ_λ is proportional to the ratio of the shunt impedance r_λ to the quality factor Q_λ

$$\chi_\lambda = \frac{\omega}{4} \frac{r_\lambda}{Q_\lambda} \tag{10}$$

and, practically, can be calculated by the numerical code URMEL. Note that URMEL defines r_λ in such way, that a factor 1/2 has to be used in Eq. (10) instead of the factor 1/4.

For modes with very high Q–factor and excited by a series of bunches of length σ and spacing s_B, $Q \gg s_B k$ the average energy loss of a test particle can be quite different from that estimated above, because of resonance of the field induced by all bunches which go ahead of the test particle. This changes the loss by the factor[8] $F(ks_B/2Q,\ ks_B)$, where s_B is the bunch spacing, and

$$F(x,y) = \frac{\sinh\ x}{\cosh\ x - \cos\ y} \tag{11}$$

The transverse δ-function wake gives the average transverse kick caused by the wake field of a point-like bunch:[1]

$$W_\perp^\delta = \frac{1}{Q}\int_{-\infty}^{\infty} dt v_s (\vec{E}_\perp(\vec{r}, z = vt - s, t) + (\vec{v} \times \vec{B}(\vec{r}, z = vt - s, t))_\perp) \tag{12}$$

where $\vec{E}$ and $\vec{B}$ also depend on the offset $\vec{r}\,'$ of the exciting bunch. In a modal analysis it can be expressed as

$$W_\perp^\delta = 2\sum_\lambda \chi_\perp^\lambda(\vec{r}, \vec{r}\,') \sin(\omega_\lambda s/v) \tag{13}$$

in analogy to Eq. (7). Analogous to Eq. (8) the transverse loss $\chi_\perp^\lambda$ is given by the eigenfunctions of the modes:

$$\chi_\perp^\lambda(\vec{r}, \vec{r}') = \frac{c}{\omega_\lambda}\nabla_\perp \frac{V_\lambda(\vec{r})\ V_\lambda^*(\vec{r}\,')}{4U_\lambda} \tag{14}$$

The Panofsky–Wenzel theorem relates the longitudinal and transverse wakes for a given mode:

$$\frac{\partial}{\partial s} W_{\perp,\lambda}^\delta = \nabla_\perp W_{l,\lambda}^\delta \tag{15}$$

The modal analysis described above is useful for narrow-band impedance for frequencies close to the beam pipe cut-off. In the high-frequency extreme, as for short CEBAF bunches, the impedance is a smooth function of frequency. This indicates that a large number of modes becomes important, so that modal analysis is not adequate. Unfortunately, rigorous results on the high frequency dependence of impedance have not been obtained. The Vainshtein-Sessler optical resonator model[4] predicts that longitudinal impedance decreases with frequency as $\omega^{-3/2}$. Recent results[6],[7], however, give $\omega^{-1/2}$ dependence. These results imply that for a single pill-box cavity with width g and attached tubes with radius a, the impedance in high frequency limit is:

$$Z_l(\omega) = \frac{Z_0}{2\pi}\sqrt{\frac{g}{\pi a}}\frac{1}{\sqrt{ka}} \tag{16}$$

If the high-frequency tail is dominant, Eq. (5) gives for the energy loss k_l

$$k_l(\sigma) = \frac{\Gamma(1/4)Z_0 c}{4\pi^2 a}\sqrt{\frac{g}{\pi\sigma}}\,, \qquad \frac{\Gamma(1/4)}{\pi} = 1.154 \tag{17}$$

This formula was discussed also by K. Bane.[8] We checked Eq. (17) with the code TBCI with parameters chosen to be close to CEBAF parameters for (1) the fundamental power couplers (a=3.5 cm, g=2.5 cm, cavity radius b=5.5 cm), (2) the higher order mode couplers (a=3.75 cm, g=3.75 cm, b=5.5 cm), and (3) gate valves (a= 1.75 cm, g= 2 cm, b= 3.5 cm). In Fig. 1a, we show TBCI data along with the values obtained from Eq. (17) for the case of the fundamental power coupler. For all three cases the dependence of the average loss vs rms bunch size in the range $\sigma = 0.75mm - 1.5mm$ corresponds to Eq. (17) and numerically agrees with Eq. (17) within 10% accuracy. See, for example, the result for the first case in Fig. (1).

The transverse impedance is built from a large number of modes, each of which satisfy the Panofsky–Wenzel theorem. If the longitudinal impedance of the transverse deflecting mode scales as $\omega^{-1/2}$, then the transverse loss factor $k_\perp = < W_\perp^s/r >$ for a pill-box cavity with attached tubes, averaged as in Eq. (2), may be written as follows:

$$k_\perp = \frac{1}{a^3}\sqrt{\pi g \sigma} \tag{18}$$

The numerical factor is determined by arguments analogous to that of Heifets and Kheifets.[7] Calculations with TBCI confirmed Eq. (18) with the same accuracy and for the same parameters that were used to check longitudinal loss. See Fig. 1b. Equations (17) and (18) can be used to obtain losses for CEBAF short bunches, for which direct simulations with the present version of TBCI are impossible.

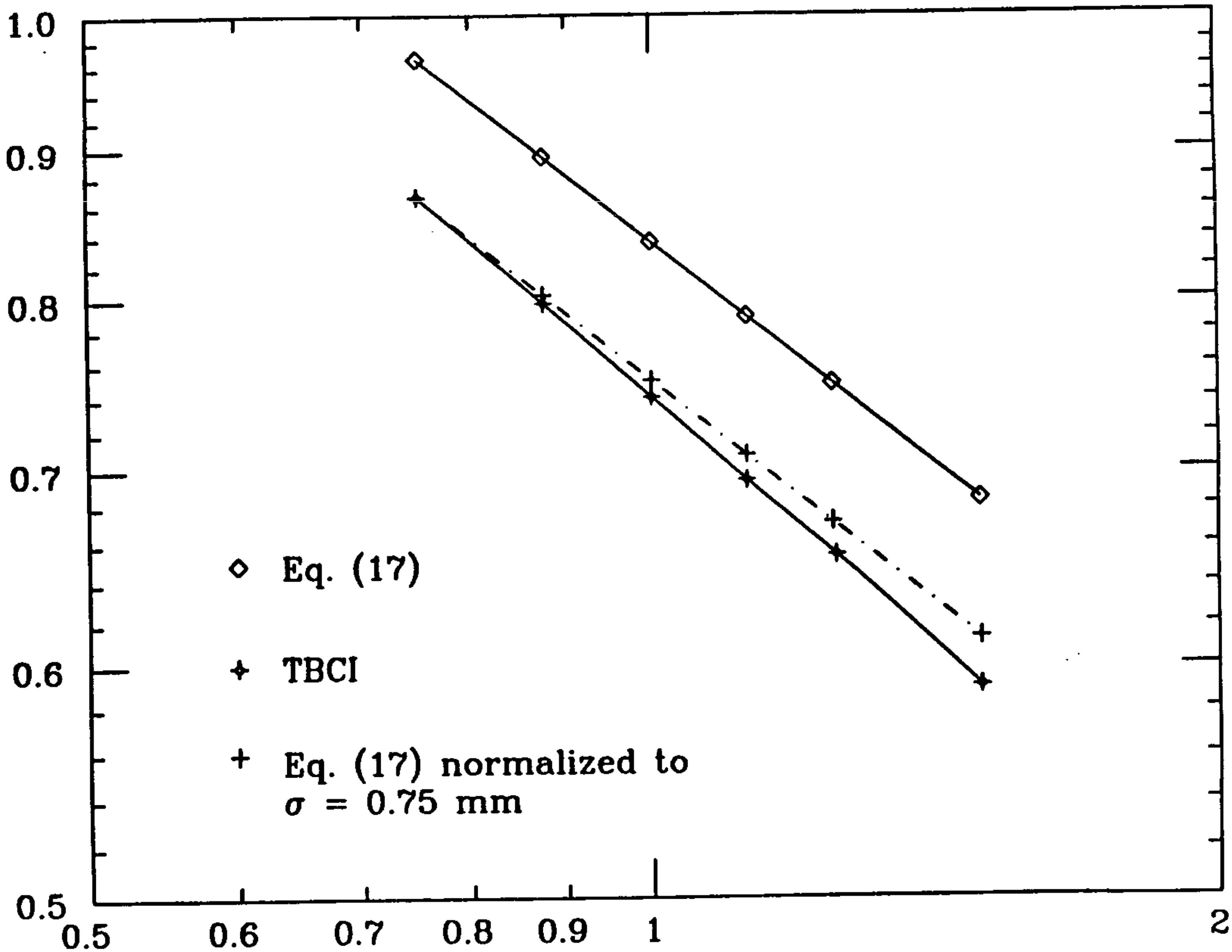

Fig. 1a. The longitudinal loss factor (V/pC) vs. σ (mm) for the fundamental power coupler.

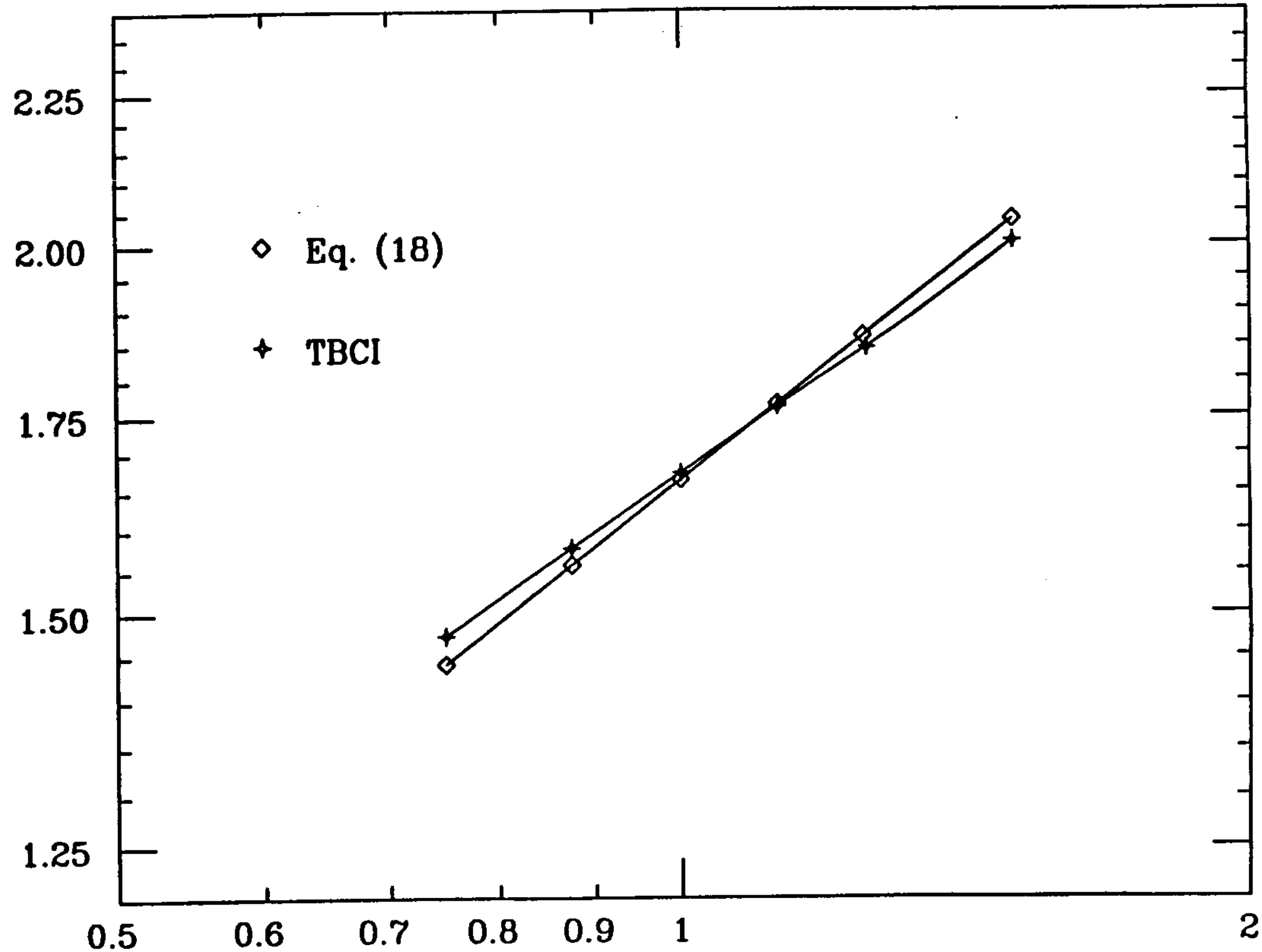

Fig. 1b. The transverse loss factor (V/pC/m) vs. σ (mm) for the fundamental power coupler.

For a wider range of parameters, in particular, for very long cavities, Eqs. (17) and (18) are not applicable. In this case we can expect transition to formulas for impedances generated by a single discontinuity (a "jump" or a "step") of the beam pipe radius. The energy loss per particle for a step was given by V. Balakin and A. Novokhatsky[9] and later studied in a semianalytical model.[11] If a particle enters a narrow pipe the impedance is negligibly small. Substantial impedance is generated only when a bunch traverses from the narrow side of the step to the wide side. In this case, the impedance is approximately independent of frequency:

$$Z(\omega) = \frac{Z_0}{\pi} \ln \frac{b}{a},$$

where $Z_0 = 377$ ohm is the impedance of free space, b and a are the radii of the wide and narrow pipes, respectively. Notice that this frequency independent impedance corresponds to a δ-function wake $W^{\delta}(s)$.

The behavior for the average losses in this case is

$$k_l = \frac{2}{\sigma\sqrt{\pi}} \ln b/a, \tag{19}$$

Conjectures along the lines which yielded equation (18) imply that

$$k_{\perp} = \frac{2}{a^2\sqrt{\pi}} \ln \frac{b}{a} \ln \frac{b}{\sigma}. \tag{20}$$

These formulas depend on both radii, in contrast to Eqs. (17) and (18). The transition from the regime of a cavity to the regime of a step depends on whether or not the signal from the wall of a cavity can reach a bunch while it is within the cavity. For $a \approx b$, this transition is defined by the ratio of the width of the cavity g to the parameter $(b-a)^2/2\sigma$. If g is bigger than this parameter, the situation is similar to that for a step, otherwise Eqs. (17) and (18) are applicable. We checked this statement numerically with TBCI. The dependence of the losses on the pill-box cavity radius b, given in Fig. 2, clearly indicates the transition from one regime to another at

$$p = 2\sigma g/(b-a)^2 \approx 1 \tag{21}$$

The more general formula for the transition parameter p which coincide with Eq. (21) for $g \gg a \approx b$ was given by Wilson.[18] Numerically, the b-dependence is in agreement with Eqs. (17-20). Figure 3 shows dependence of the losses on rms bunch size. Equation (19) is compared with TBCI results in Fig. 4 for $p \gg 1$. Unfortunately, the available version of TBCI does not enable a more thorough verification of Eqs. (17-20).

If a beam pipe is tapered, the losses become smaller. There are no analytic results available on the effect of tapering. We studied the effect numerically with TBCI for a long pill box cavity with attached tubes, with parameters a=0.01 m, b=0.03 m, and g=1.0 m. The transition from one radius to another was described by the function

$$r(z) = 0.5\{b + a - (b-a)\tanh(\frac{|z| - g/2}{\delta})\}$$

with δ in the range from 0.0 to 0.10 m. The result for the longitudinal and transverse wakes, averaged with a Gaussian bunch, are shown by the solid line in Fig. 5 for σ = 0.6 cm and in Fig. 6 for σ = 0.25 cm. The slope scales with radius a as $(1/a)\ln(b/a)$. The δ-dependence is steeper for larger σ. The maximum value for the longitudinal wake, which affects the energy spread, is about two times larger than the average value and it depends on δ similarly to the loss factor. The same is true for the maximum value of the transverse impedance. Figures 5 and 6 show that tapering can decrease loss factors by several times.

The contradiction between the optical resonator model and the results for a single cavity might be understood if the result of the optical resonator model is valid for a periodic structure only. Today this problem has not been rigorously solved. The related problem, of how the impedance scales with the number of adjacent cavities, has not been answered either. We compared the losses for a 6-convolution bellows with a pitch 2.1 mm, a=1.74 cm, and b=2.26 cm, with the total loss factor of six independent single pill-box cavities with attached tubes. Each cavity simulates a single convolution. The losses of the bellows, given by TBCI for σ=0.25

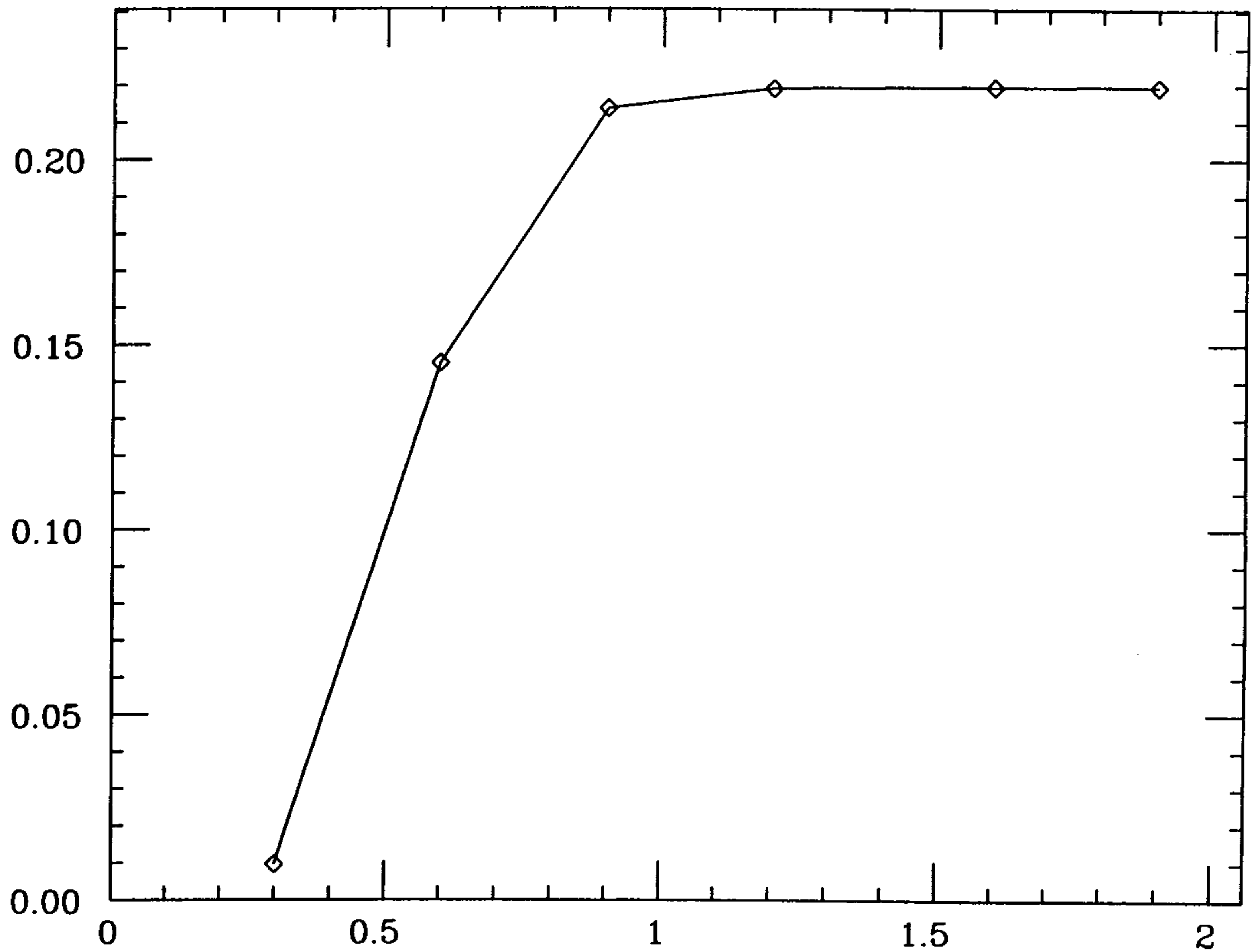

Fig. 2a. Transition from the regime of a cavity to the regime of a step. The longitudinal loss factor (V/pC) vs b (m)

σ=0.06, a=0.25, g=6.0, L_{tot}=7.0

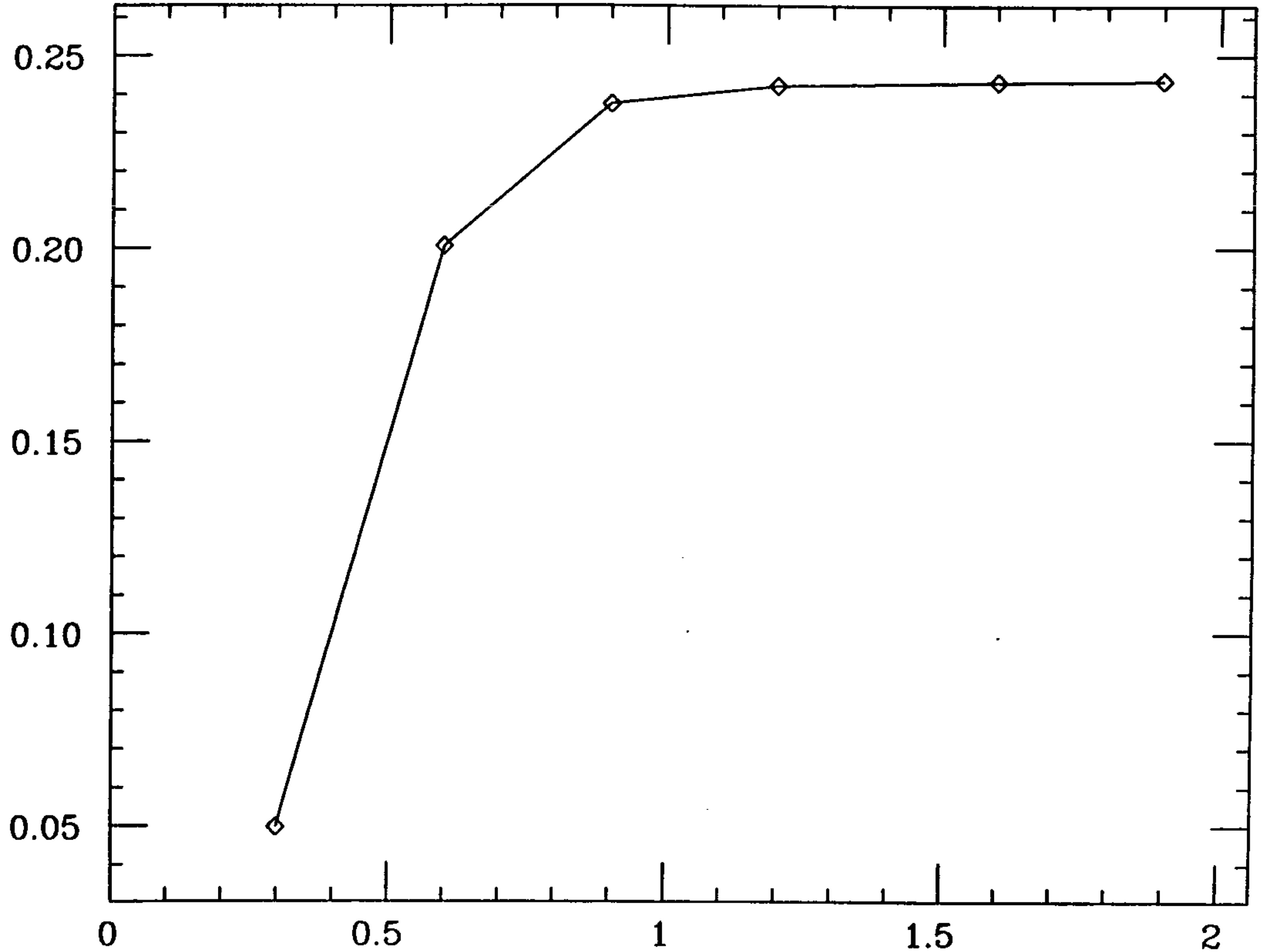

Fig. 2b. The transition from the regime of a cavity to the regime of a step. The transverse loss factor (V/pC/m) vs. b (m)

σ=0.06, a=0.25, g=6.0, L_{tot}=7.0

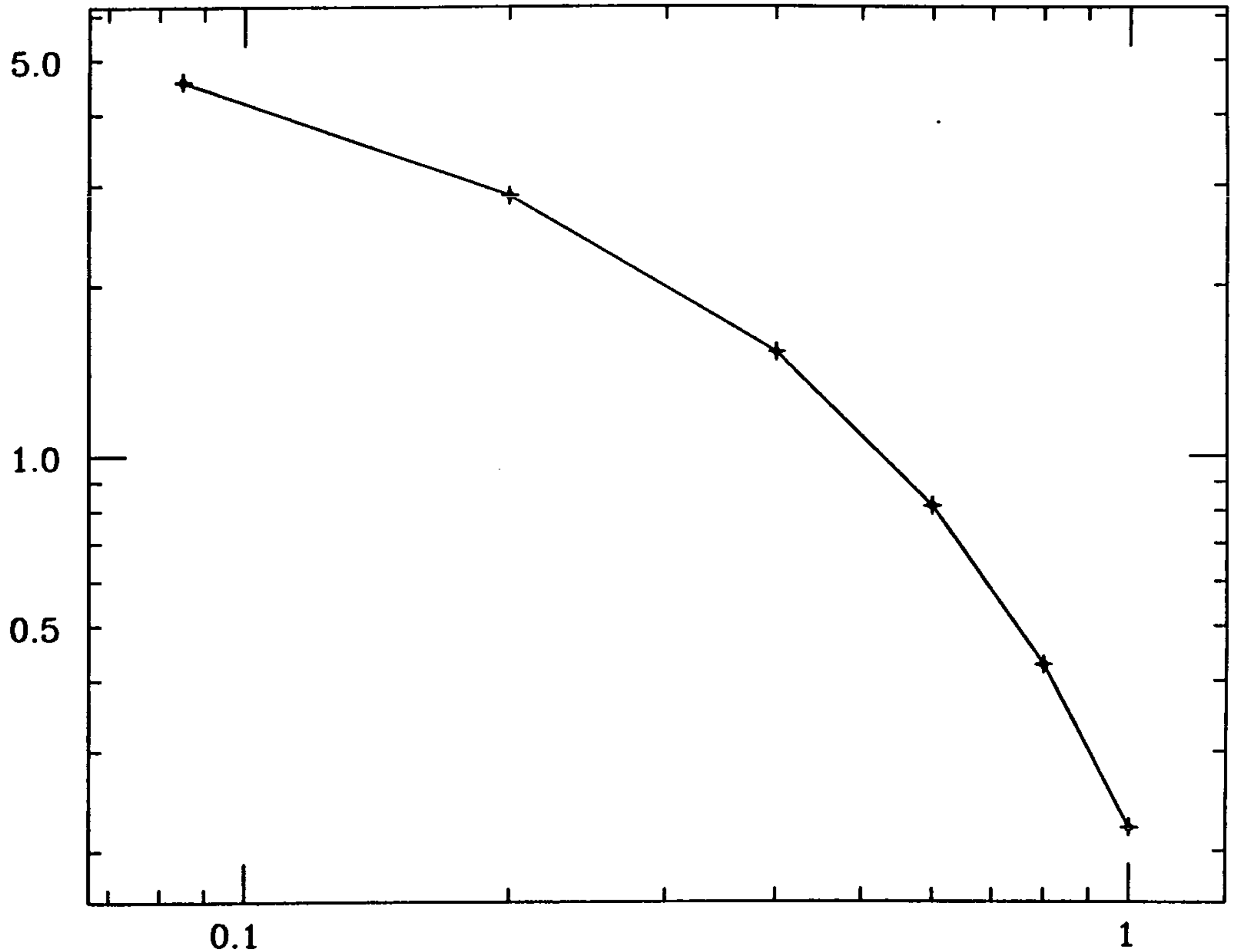

Fig. 3a: The transverse loss factor (0.01 V/pC) vs σ (m)

$a=1.25,\ b=2.50,\ g=8.8,\ L_{tot}=12.0$

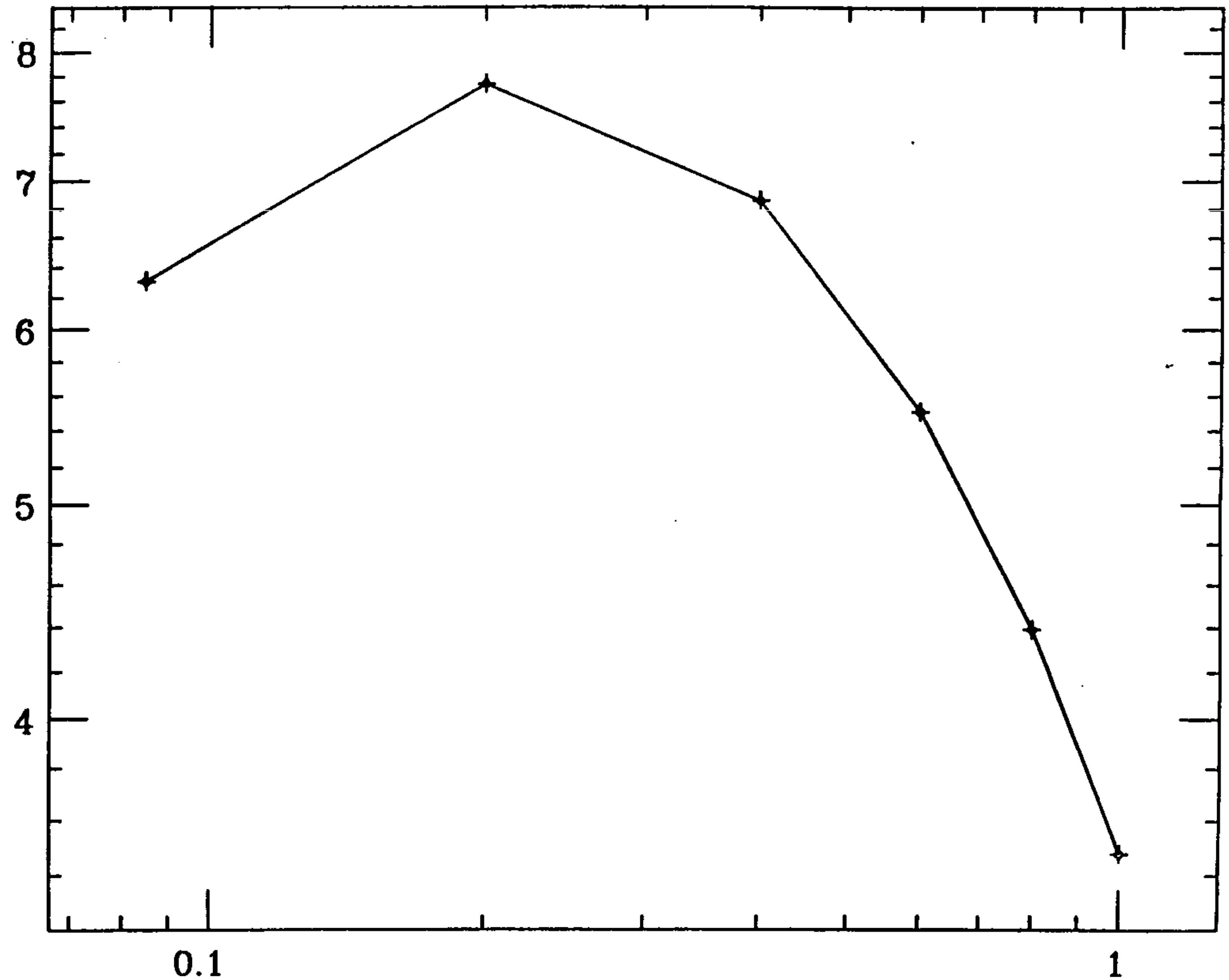

Fig. 3b: Transverse loss factor (0.001 V/pC/m) vs σ(m)

a=1.25, b=2.50, g=12.5, L_{tot}=15.5

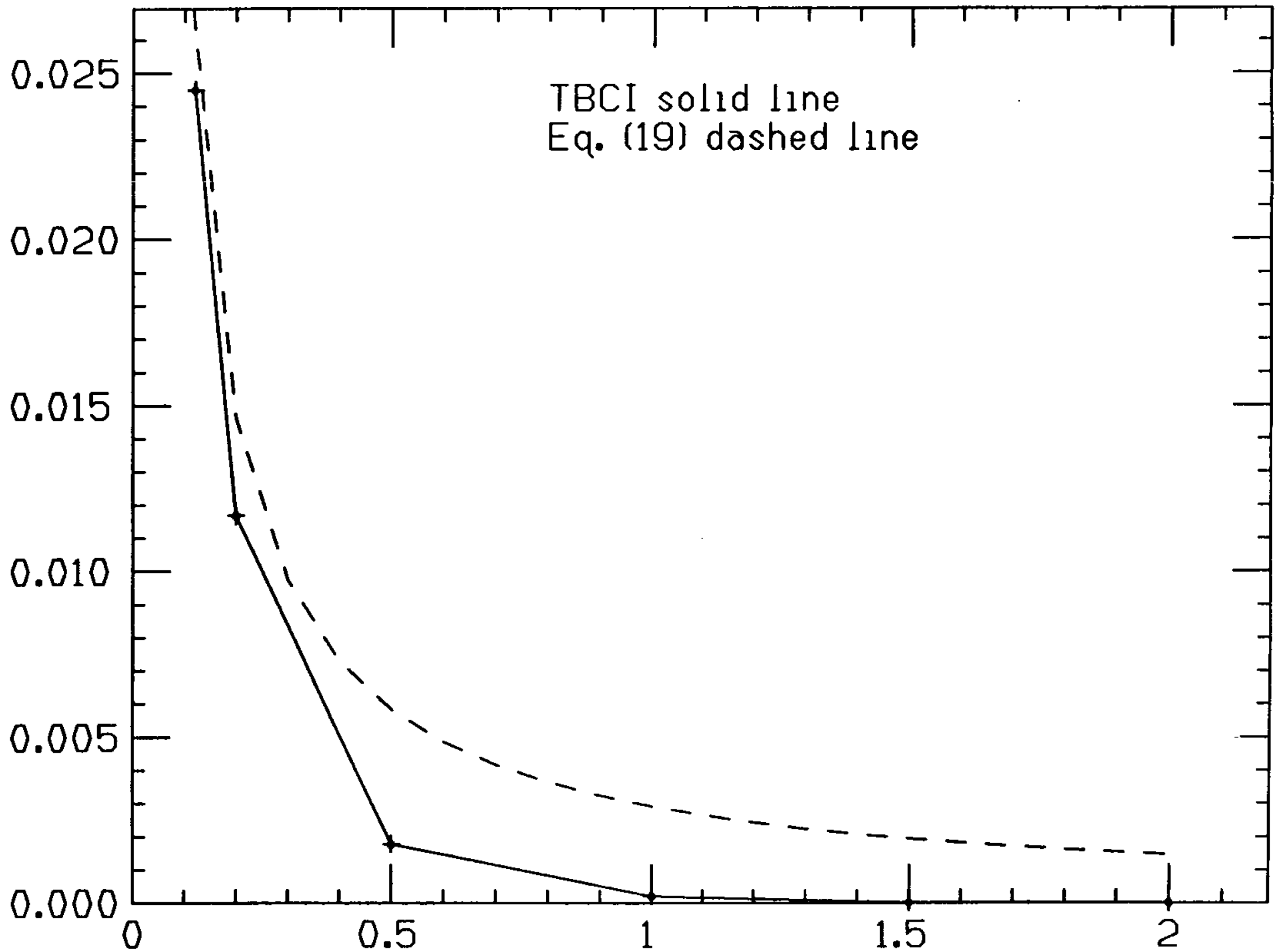

Fig. 4a: The longitudinal loss factor (V/pC)
for a step vs. σ(m)
$a=1.5$, $b=2.0$, $g=20.0$, $L_{tot}=30.0$

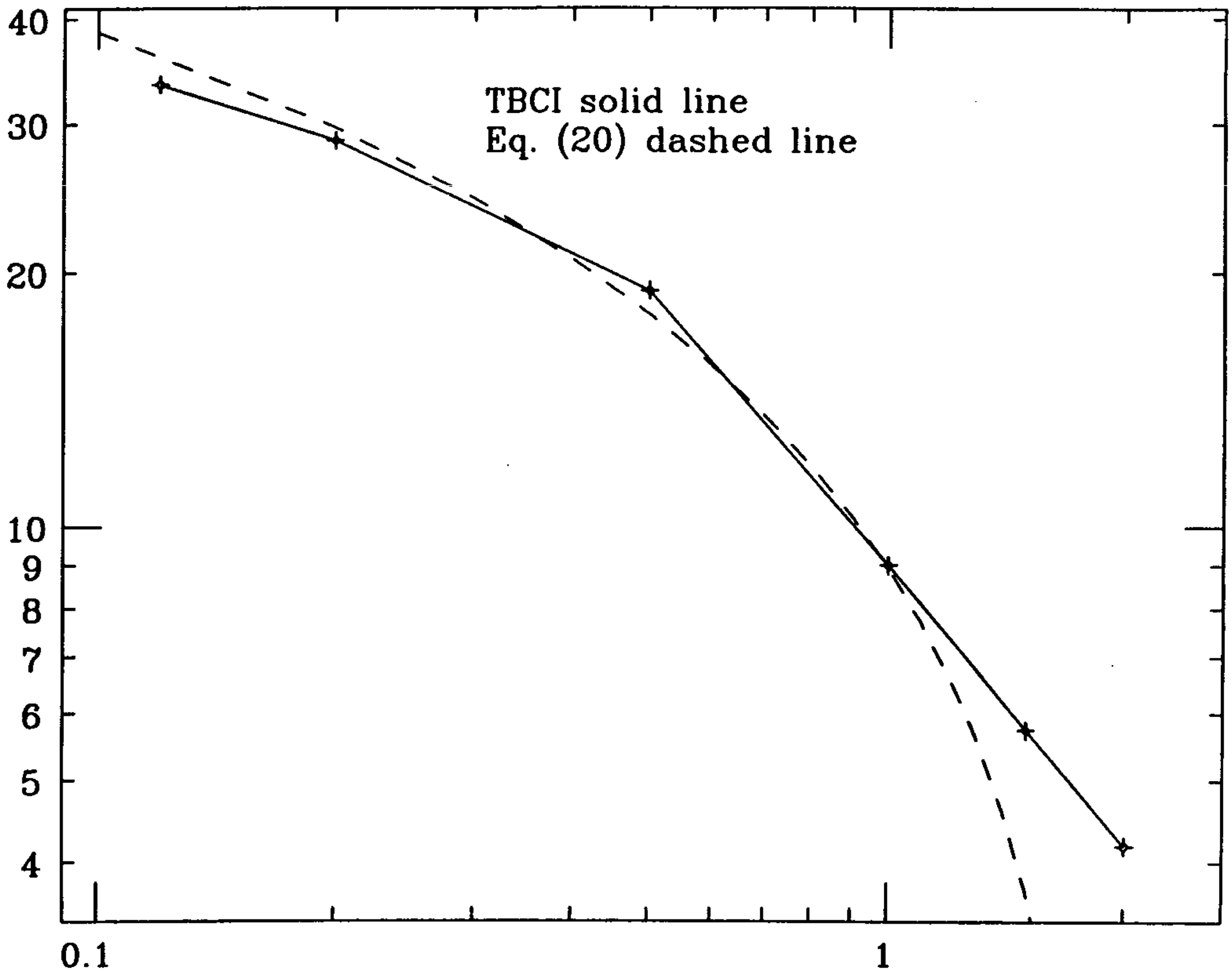

Fig. 4b: The transverse loss factor (0.0001 V/pC/m)
for a step vs. σ(m)
$a=1.5$, $b=2.0$, $g=20.0$, $L_{tot}=30.0$

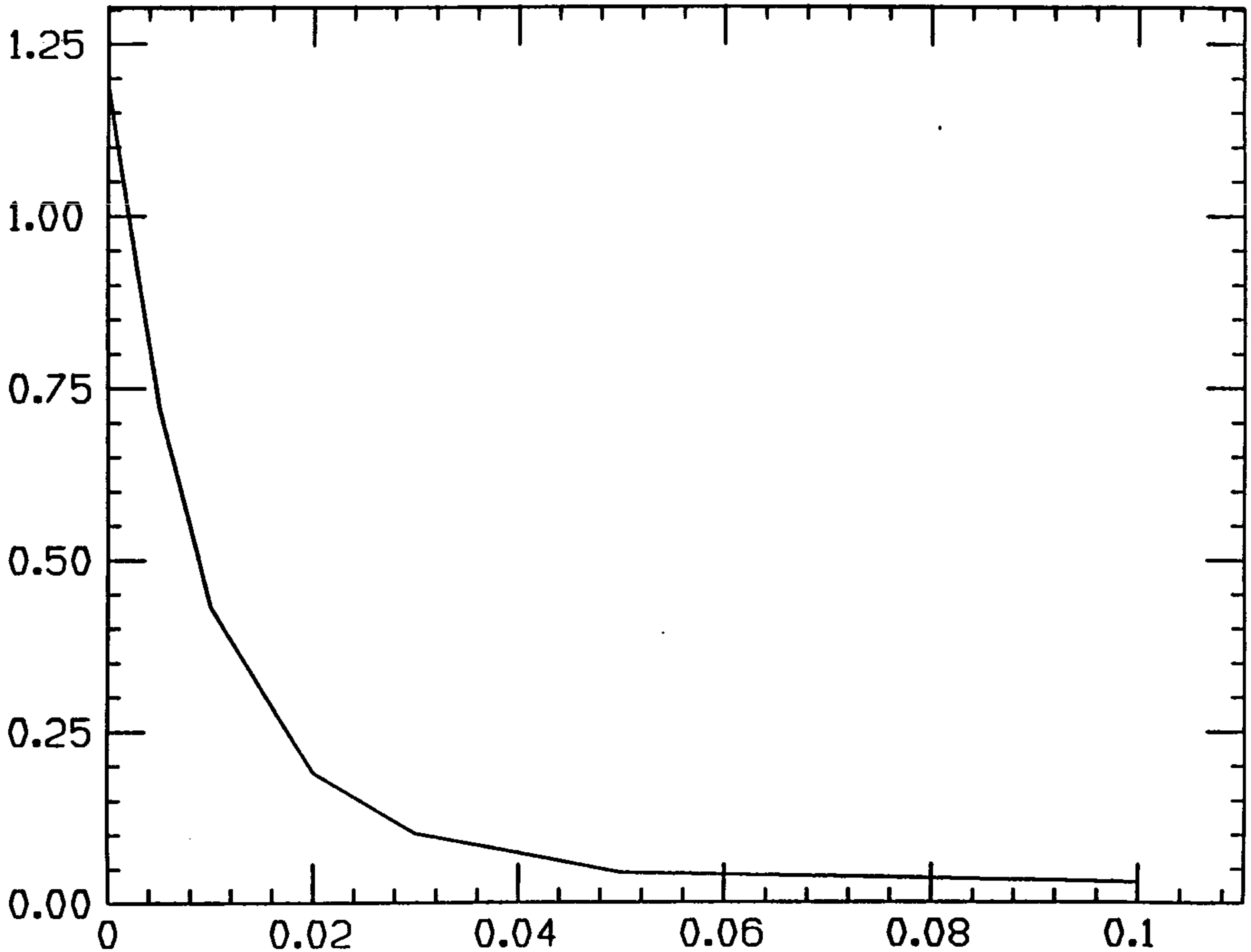

Fig. 5a: The longitudinal loss factor (V/pC)
vs width of a step (m)
a=.01, b=.03, σ=0.006, δ=0.50

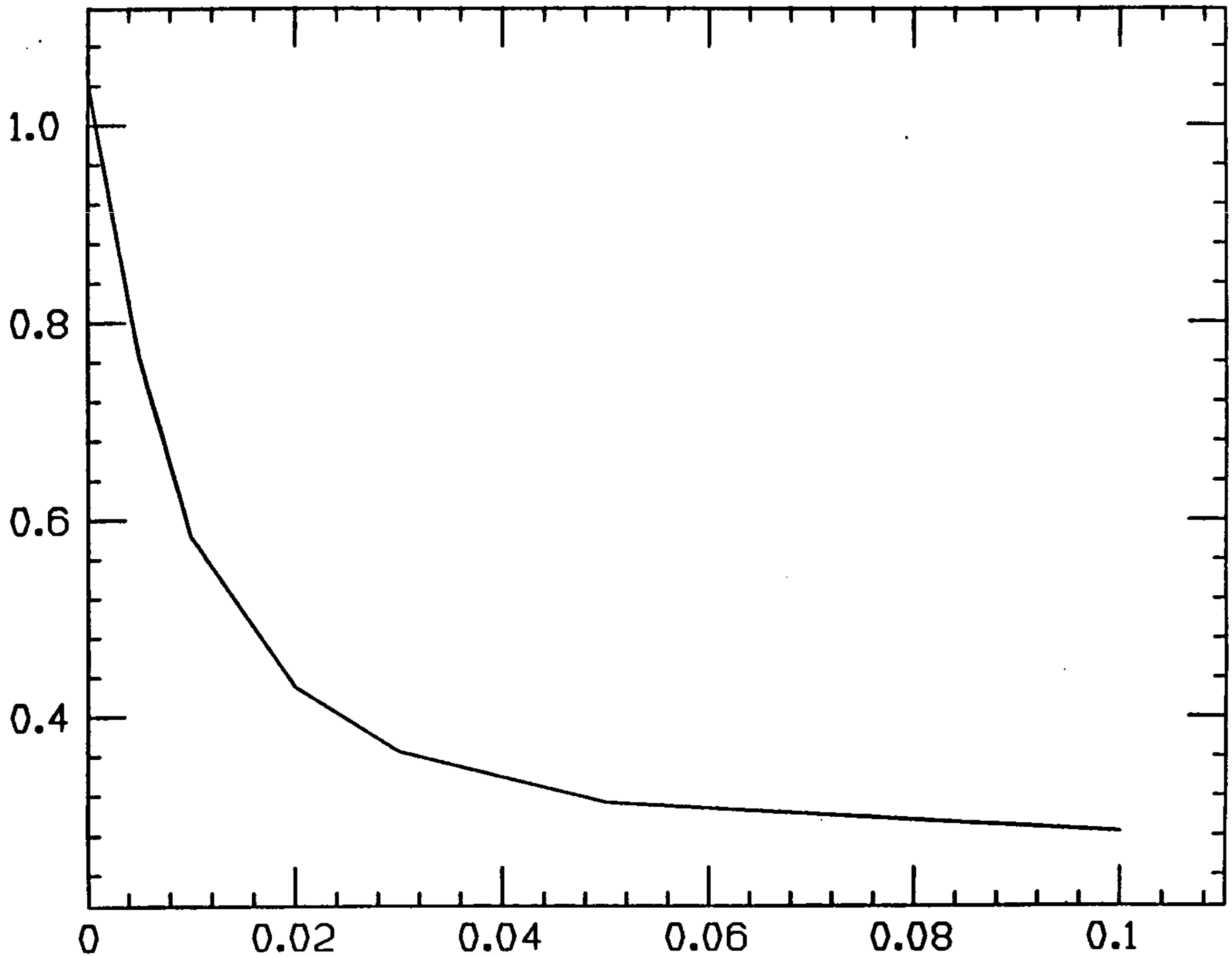

Fig. 5b: The transverse loss factor (V/pC)
vs width of a step (m)
a=.01, b=.03, σ=0.006, δ=0.50

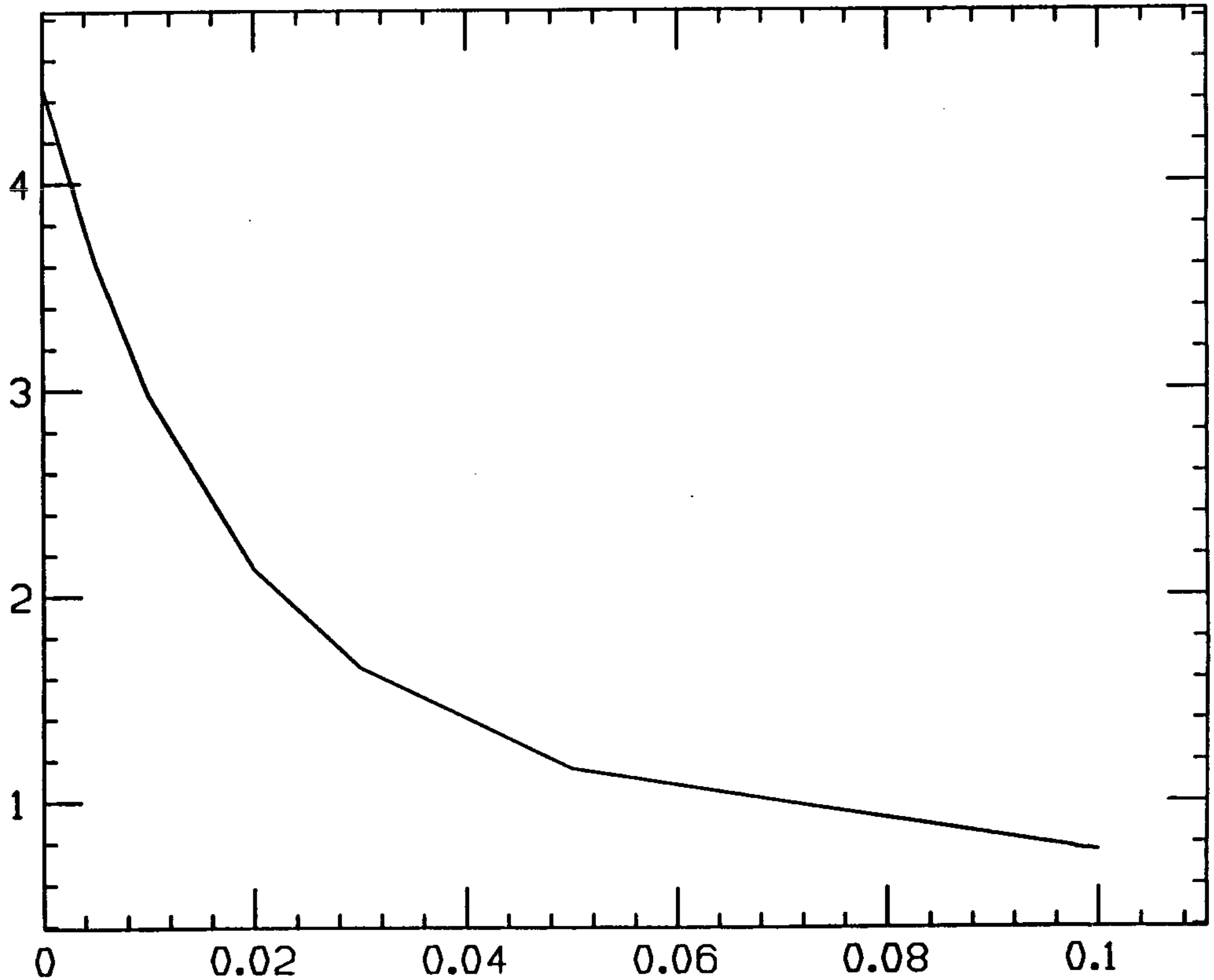

FIG. 6a: The longitudinal loss factor (V/pC)
vs width of a step (m)
$a=.01$, $b=.03$, $\sigma=0.0025$, $\delta=0.50$

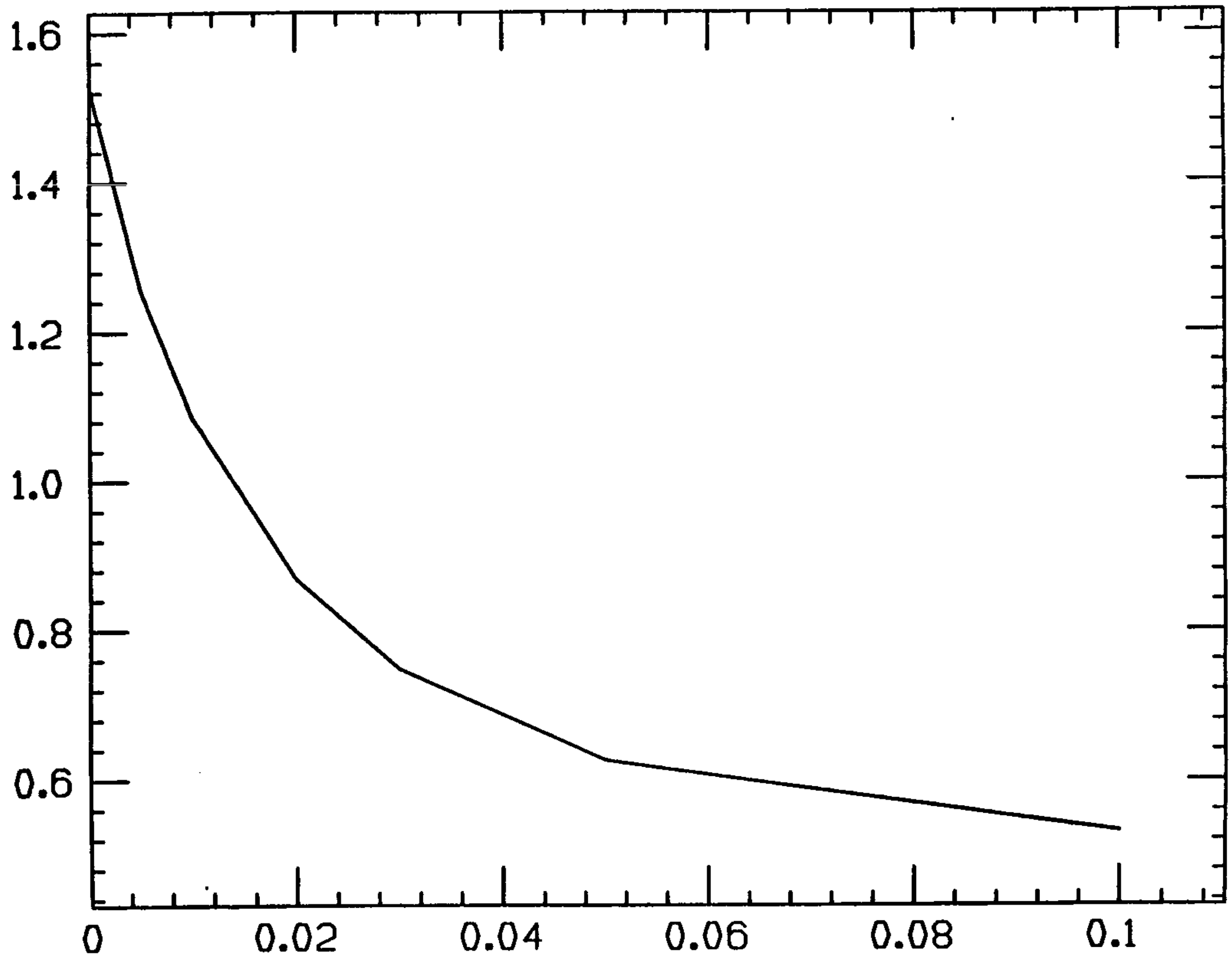

Fig. 6b: The transverse loss factor (V/pC/m)
vs width of a step (m)
$a=.01$, $b=.03$, $\sigma=0.0025$, $\delta=0.50$

mm, are $k_l = 3.56$ V/pC and $k_\perp = 8.28$ V/pC/m. Equations (17) and (18) give correspondingly 5.81 V/pC and 13.05 V/pC/m. Hence, longitudinal and transverse losses for a bellows with 6 convolutions are about 40% less than those for 6 independent cavities.

Conclusion

The main results of the paper are given in Eqs. (17-21). The loss parameters for a cavity and a step given in these formulas are in good agreement with numerical results with TBCI. These expressions provide simple but reasonable estimates of the loss factors of short bunches.

Acknowledgement

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